UDC 330.43 + 519.862

DOI: https://doi.org/10.32782/2520-2200/2020-1-44

Kozytskyi Valerii
Ivan Franko National University of Lviv
Beregova Galyna
Lviv Politechnic National University
Pabyrivska Nelya
Lviv Politechnic National University

Козицький В.А.
кандидат фізико-математичних наук, доцент,
доцент кафедри математичної економіки та економетрії
Львівського національного університету імені Івана Франка
Берегова Г.І.
кандидат фізико-математичних наук, доцент,
доцент кафедри обчислювальної математики та програмування
Національного університету «Львівська політехніка»
Пабирівська Н.В.
кандидат фізико-математичних наук, доцент,
доцент кафедри вищої математики
Національного університету «Львівська політехніка»

SYSTEM DYNAMIC NONLINEAR MODELING OF PRICE STABILITY

СИСТЕМНЕ ДИНАМІЧНЕ НЕЛІНІЧНЕ МОДЕЛЮВАННЯ ЦІНИ СТАБІЛЬНОСТІ

The article investigates the dynamic properties of price indexes based on the system dynamics approach that is a simulation modeling approach to monetary policy analysis and design. It applies to dynamic problems that describe complex economic and social systems including information and material delays as well as feedback and circular causality. The simulation nonlinear model takes into account the nonlinear process of price adjustment. The simulation results showed that the behavior of price indexes could change dramatically that was driving by small change in initial conditions and parameters of system. It has been obtained that the inflation process looked often as unpredictable and chaotic system although it had underlying sources and its driving forces did not demonstrate pure randomness. The price dynamics should be considered in a system and dynamic view and it is important to estimate the direction and magnitudes of unemployment and inflation responses on positive and negative economic shocks.
Key words: system dynamics, price, simulation model, nonlinearity, stability properties, equilibrium.

Як і багато інших країн з перехідною економікою, Україна пережила роки економічних спадів, що супроводжувались зростанням безробіття та високою інфляцією. Подолання інфляції в Україні є надзвичайно важливою проблемою, оскільки її темпи часто були високими, а відтак інфляційні очікування є постійними. Інфляційний процес часто виглядає як непередбачувана і хаотична система, хоча він має підґрунтя та рушійні сили, які не є повністю випадковими. Тому процеси стабільності ціноутворення необхідно розглядати системно та враховувати їх динамічний характер. У статті досліджено динамічні властивості індексів цін на основі підходу системної динаміки, який є методом імітаційного моделювання та дає змогу провадити аналіз та розробку заходів монетарної політики. Досліджено динамічні проблеми, що описують складні економічні та соціальні системи, які характеризуються інформаційними та матеріальними затримками, а також зворотними зв'язками та циклічністю. Побудована імітаційна модель враховує нелінійний процес коригування цін. Результати моделювання показали, що поведінка цінових індексів може кардинально змінитися за невеликих змін у початкових умовах та параметрах системи. Встановлено, що інфляційний процес часто виглядає як непередбачувана і хаотична система, хоча він має основні джерела та чинники, які зумовлюють його розвиток, а рушійні сили не демонструють чистої випадковості. Отож, важливо оцінити напрямок та масштаби відгуків безробіття та інфляції на позитивні та негативні економічні збурення. За невеликого зміщення економічних передумов до величини, яка дуже близька до попередньої, спостерігається зовсім інша динаміка системи цін. Важливість визначення рівнів та темпів інфляції

досліджується за допомогою аналізу структури моделі та її динамічних властивостей. Хоча дискретний погляд, орієнтований на окремі події та рішення, повністю сумісний з ендогенною перспективою зворотного зв'язку, підхід системної динаміки підкреслює неперервність процесу ціноутворення. Модель відображає динамічні закономірності, що лежать в основі процесу формування цін, зосереджується не на дискретних рішеннях, а на структурі політики, що лежить в основі цих рішень.

Ключові слова: системна динаміка, ціна, імітаційна модель, нелінійність, властивості стійкості, рівновага.

В статье исследованы динамические свойства индексов цен на основе подхода системной динамики, что позволяет проводить анализ и разработку мероприятий монетарной политики. Исследовано динамические проблемы, описывающих сложные экономические и социальные системы, характеризующиеся информационными и материальными задержками, а также обратными связями и цикличностью. Построена имитационная модель, учитывающая нелинейный процесс корректировки цен. Результаты моделирования показали, что поведение ценовых индексов может кардинально меняться при небольших изменениях в начальных условиях и параметрах системы. Установлено, что инфляционный процесс часто выглядит как непредсказуемая и хаотичная система, хотя обусловлен детерминированными факторами, а его движущие силы не демонстрируют чистой случайности. Динамику цен следует рассматривать в системном и динамическом отношении, оценивая направление и величину отзывов уровня безработицы и инфляции на положительные и отрицательные экономические шоки.

Ключевые слова: системная динамика, цена, имитационная модель, нелинейность, свойства устойчивости, равновесие.

Introduction. As many other transition economies, Ukraine went through years of economic decline with rising unemployment and high inflation. The problem of inflation in Ukraine is extremely important because it was increasing over years and price growth rate was high. The inflation process looks often as unpredictable and chaotic system although it has underlying sources and its driving forces are not fully random. Therefore, price stability should be considered in a system and dynamic view. H. Hwarng and X. Yuan (2014) found the chaotic phenomena, chaos amplification and other nonlinear behaviors in supply chain systems. They derived that the effective inventory at different supply chain levels demonstrated various chaotic dynamics that depended on specific deterministic demand settings [1]. Scientists showed different asymmetric and nonlinear properties in behavior main macroeconomic indicators that evaluated economic development and stability in Ukraine as well as in European countries [2; 3].

Literature review. M. Dusza (2017) focused on financial crisis emphasizing the crisis in the banking system, chaos on the foreign exchange market as well as troubles on capital markets. He proved that the financial pyramids could be the main source and the element of financial instability, social unrest and wars that had unpredictable influence on financial markets and revealed chaotic dynamics [4]. Scientists also used a wide variety of modern nonlinear econometric approaches to investigate the dynamic asymmetric peculiarities in behaviour of macroeconomic factors of labor market and emphasized the important of estimation for the direction and magnitudes of unemployment and inflation responses on positive and negative economic shocks [5; 6].

V. Anishchenko, T. Vadivasova, G. Okrokvertskhov and G. Strelkova (2003) investigated the correlation and spectral peculiarities for chaotic system and showed a range of the self-sustained oscillations with different types. They indicated the impact of noise on chaotic systems [7]. M. Oliskevych and V. Tokarchuk (2018) proved the existence of two different regimes in dynamics of unemployment rate during the different periods of economic cycles and its significant dependence on economics shocks that occurred in European area [8]. D. Yan, X. Ma and T. Li (2018) studied the long-term competition in a recycling price game model and described the Nash equilibrium point properties in the corresponding stable region. It was showed that the stability of the price system is substantially affected by adjustment speed of the recycling price, the sensitivity of consumers and the price cross-elasticity between manufacturers and retailers [9].

Methods. System dynamics is a computer-aided approach to policy analysis and design. It applies to dynamic problems arising in complex social, managerial, economic, or ecological systems – literally any dynamic systems characterized by interdependence, mutual interaction, information feedback, and circular causality. The approach begins with defining problems dynamically, proceeds through mapping and modeling stages, to steps for building confidence in the model and its policy implications (Figure 1).

Simulation of such systems is easily accomplished by partitioning simulated time into discrete intervals of length *dt* and stepping the sys-

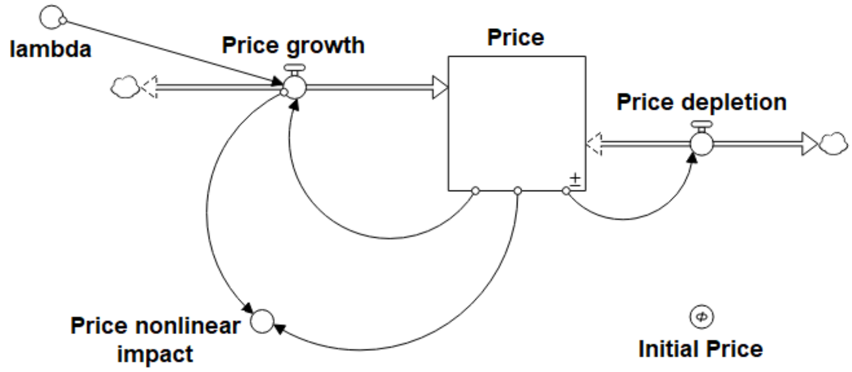


Figure 1. The basic structure of system dynamics model for price adjustment

Source: constructed by the authors

tem through time one *dt* at a time. Each state variable is computed from its previous value and its net rate of change $x'(t)$. The computation interval *dt* is selected small enough to have no discernible effect on the patterns of dynamic behavior exhibited by the model. In more recent simulation environments, more sophisticated integration schemes are available (although the equation written by the user may look like this simple Euler integration scheme), and time scripts may not be in evidence [10].

The basic price equation can be represent in the form

$$p(t+1) = f(p(t)) = \lambda\, p(t)\,(1 - p(t)),\ 0 \le \lambda \le 4. \quad (1)$$

Set the equilibrium points, where $p(t+1) = p(t) = p^*$ for all t. Then,

$$p^* = \lambda p^* (1 - p^*),$$

$$(p^*)^2 \lambda + p^* (1 - \lambda) = 0\ p^* [\lambda p^* + (1 - \lambda)] = 0$$

that is, we have two fixed points

$$p^*_1 = 0,\ p^*_2 = (\lambda - 1) / \lambda. \quad (2)$$

To investigate the stability, it is necessary to use a linear approximation around a fixed point. This is given by

$$p(t+1) = f(p^*) + f'(p^*)\,(p(t) - p^*).$$

However, if p^* is an equilibrium point, then $f(p^*_1) = 0, f(p^*_2) = (\lambda - 1) / \lambda$. In addition, $f'(p^*) = \lambda - 2\lambda p^*$. And so

$$f'(p^*) = \{\lambda, p^*=0;\ 2 - \lambda, p^*=(\lambda - 1) / \lambda\}. \quad (3)$$

First, consider $p^*_1 = 0$. If $0 < \lambda < 1$, then the system around this fixed point is stable. For $p^*_2 = (\lambda - 1) / \lambda$, we get

$$p(t+1) = p^* + (2 - \lambda)\,(p(t) - p^*)$$

or

$$u(t+1) = (2 - \lambda) u(t),$$

where $u(t+1) = p(t+1) - p^*$ and $u(t) = p(t) - p^*$. Therefore, the system around this fixed point is stable, if

$$|2 - \lambda| < 1, -1 < 2 - \lambda < 1, 1 < \lambda < 3.$$

Thus, the system is stable around the second fixed point for $1 < \lambda < 3$. For $0 \le \lambda < 1$, $p^*_1 = 0$ is the only equilibrium point and it is locally stable. The point $p^*_1 = 0$ is point of attraction. For $1 < \lambda < 3$, we have an equilibrium solution $p^*_2 = (\lambda - 1) / \lambda$, which changes by the parameter λ. Figure 1 represents the developing of two fixed points for different value of parameter. At $\lambda = 1$, where two solution curves intersect, stability changes from one equilibrium solution to other.

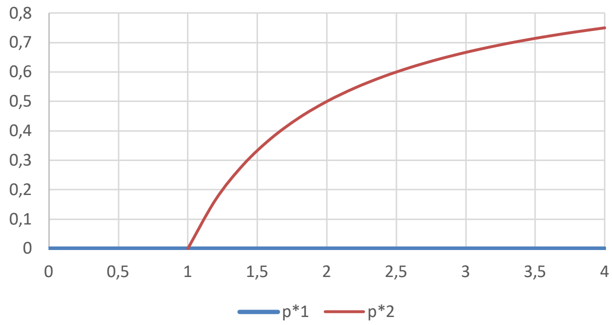


Figure 2. Fixed price level points as a function of price adjustment coefficient

Source: evaluation of the authors

In practice, the value of λ is not limited to 3. What happens when the value of λ is greater than 3 is need to be investigated deeply. We can get some idea of the problem by looking more closely at the equilibrium conditions. Given $f(p) = \lambda p\,(1 - p)$ the fixed points can be set by finding a that satisfies $a = f(a)$.

If two-cycles occurs, then the condition $a = f(f(a))$ is satisfied. When we have one cycle we can find a. Thus, we need to find where $f(a)$ cuts the 45*-line. Similarly, we can determine the values of two-cycles, if it exists, by looking for the case where $f(f(a))$ cuts the 45*-line. The situation will be different for different values of λ. The analysis of cycles is represented in Figure 3.

Results. For small price adjustment coefficient $\lambda = 0.8$ there is only one equilibrium price level

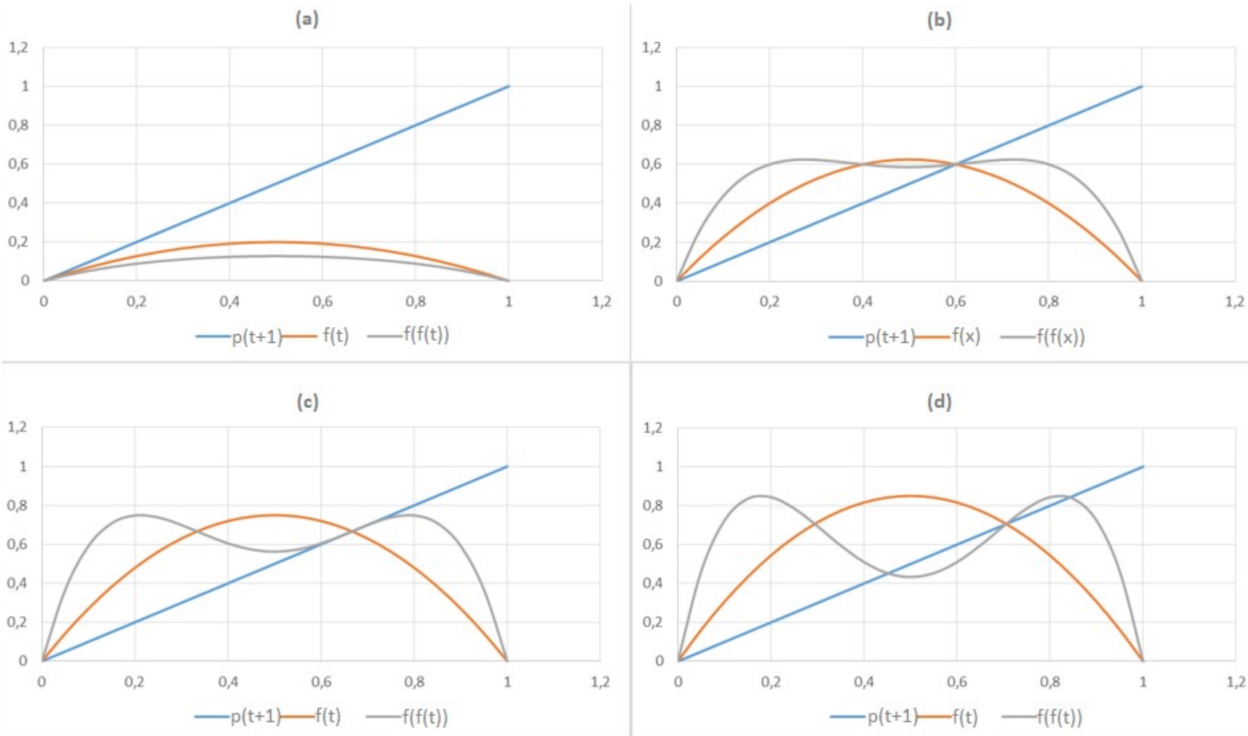


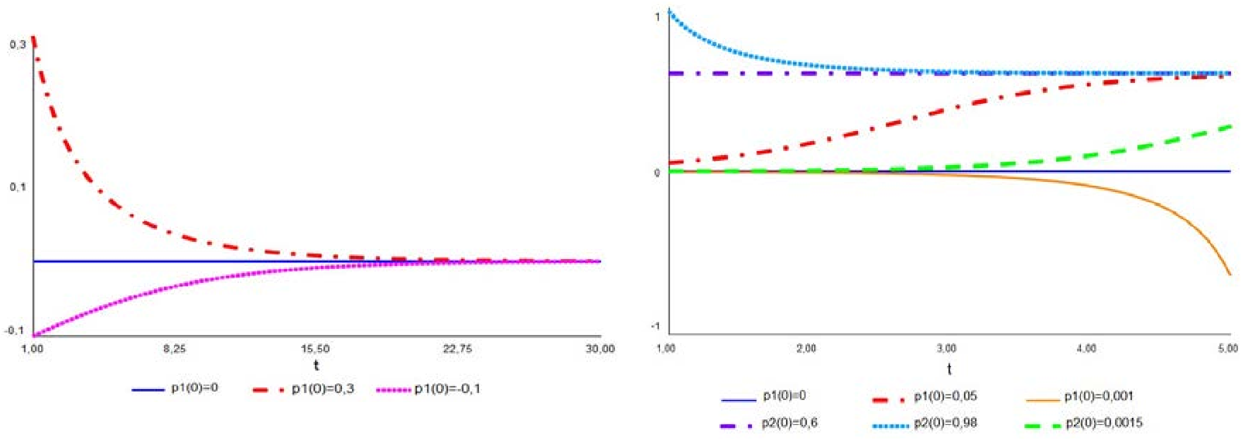
Figure 3. Price level for different types of cycles that depend on value of the price adjustment strength
Source: evaluation of the authors

$p^*_1 = 0$ (Figure 3a) since this is the only value at which $f(p)$ cuts a 45°-line. In fact, this is true for any value of λ that lies between zero and unity. This means that there are no two-cycles for this range λ. Therefore, for any initial value of p that is close enough to zero the system is attracted to the fixed point (Figure 4a). Because $f'(p^*_1) = 0.8 < 1$ the equilibrium point p^*_1 is locally stable.

For price adjustment $\lambda = 2.5$ we get $p^*_2 = (\lambda - 1) / \lambda = 0.6$. In addition from (3), we have $f'(p^*_2 = 0{,}6) = -0.5$ and since the absolute value is from 0 to 1 then $p^*_2 = 0.6$ is stable. $f(f(p))$ cuts the 45°-line only once (Figure 3b) so again no two cycles occur (Figure 4b). In fact, there is only one positive value when λ exceeds the interval $1 < \lambda < 3$.

The situation starts to change when price adjustment coefficient is equal to 3 (Figure 3c). In this case, the curve $f(f(p))$ is tangential to the 45°-line at the fixed point. The value of the fixed point is $p^*_2 = (\lambda - 1) / \lambda = 2/3$. In this case $f'(p^*_2) = -1$ and the fixed point is semistable (Figure 5).

As soon as λ goes beyond value 3, then the curve $f(f(p))$ cuts the 45°-line in three places (Figure 3d). We see that the curve $f(p)$ cuts the



a) b)

Figure 4. Price convergence to the equilibrium point (a) for λ=0.8; (b) for λ=2.5
Source: evaluation of the authors

curve $f(f(p))$ on the 45*-line and this is the central value of the three points of intersection. This value is given by $p^*_2 = 0.70588$.

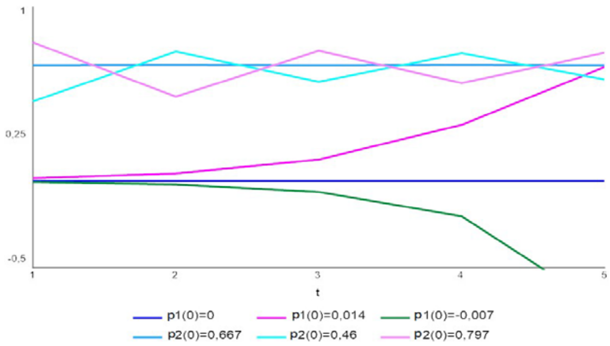
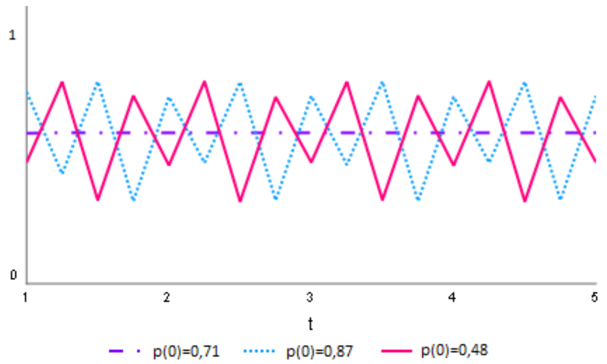


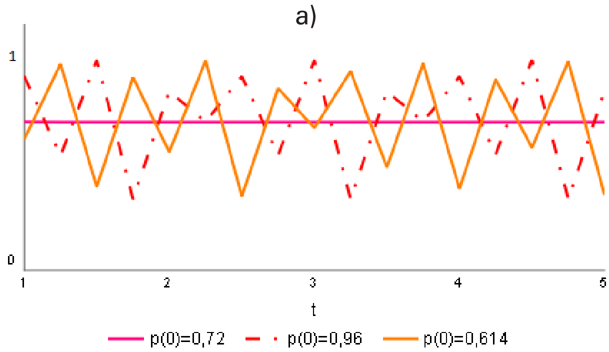
Figure 5. The dynamics of price for λ = 3 adjustment coefficient

Source: evaluation of the authors

The system is actually unstable. Exact intersection points are easy to install. The lower value is about 0.45 and the upper is about 0.84. However, when λ = 3.449, the two-cycle becomes unstable (Figure 6). This is manifested in the fact that the cycle becomes unstable and is broken into four cycles. This, in turn, is divided into eight cycles, etc. In addition, there are also odd cycles. As λ approaches 3.65, there are no regular cycles at all, and the whole picture is one of chaos. If we increase the interval of time to 300, then we can clearly see that the system behaves chaotically (Figure 6).



a)



b)

Figure 6. The price cyclic dynamics of price for λ > 3

Source: evaluation of the authors

Although there is not stochastic element, for λ = 3.94 there is a chaos in price dynamics. Table 1 represents different cases of price stability. The two-cycle ends with a value of λ, which is approximately equal to 3.57.

Table 1

Patterns for price dynamics

Type of cycles	Value of adjustment coefficient
Exchange of stability	1
Fixed point becomes unstable (2-cycles appear)	3
2-cycle becomes unstable (4-cycles appear)	3.44949
4-cycles becomes unstable (8-cycles appear)	3.54409
Upper limit value on 2-cycles (chaos begins)	3.57
First odd-cycle appears	3.6786
Cycles with period 3 appears	3.8284
Chaotic regions ends	4

Source: evaluation of the authors

Discussion. The chaotic systems are very sensitive to initial conditions (Figure 7). We set the value of adjustment to be 3.65 and investigate the price dynamics for two different initial values $p(0) = 0.1$ and $p(0) = 0.105$ that are very close to each other.

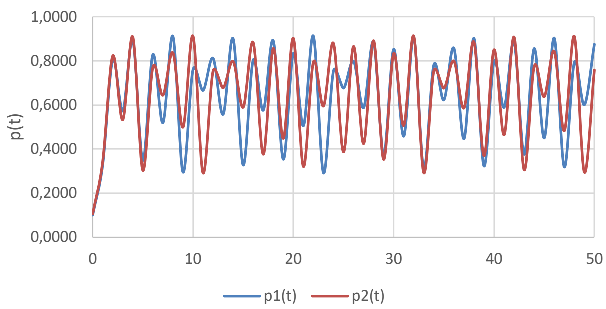


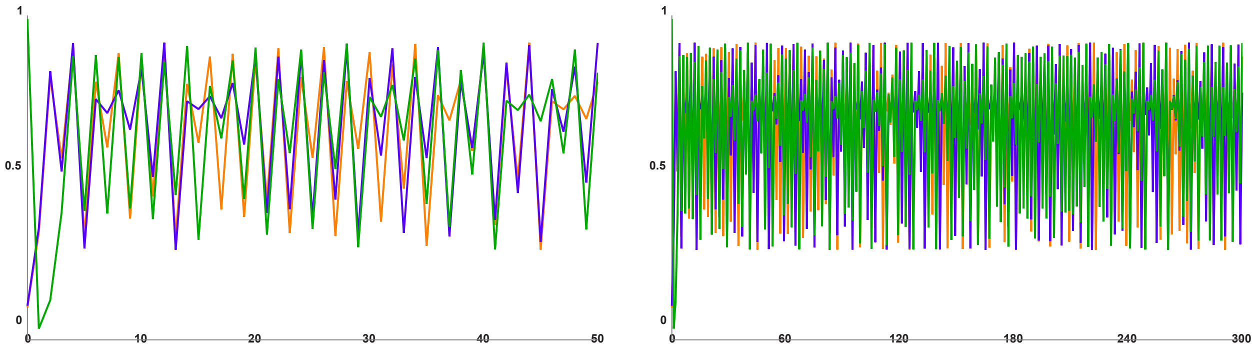
Figure 7. Price sensitivity to initial conditions

Source: evaluation of the authors

Although there is no stochastic element in the model, we obtain that the system of price behaves chaotically in the long run (Figure 8).

Another characteristic arises in the case of a series entering the chaotic region for parameter value 3.94 and $p(0) = 0.99$. In this case, although the series is chaotic, it is not purely random and exhibits sudden changes (Figure 9).

The series suddenly changes from showing oscillation to one that is almost horizontal. This change lasts for about ten periods and then, suddenly and for no apparent reason, begins to oscillate once again despite the fact the system is deterministic.



a) b)

Figure 8. The chaotic price dynamics for different initial value in the short and long run

Source: evaluation of the authors

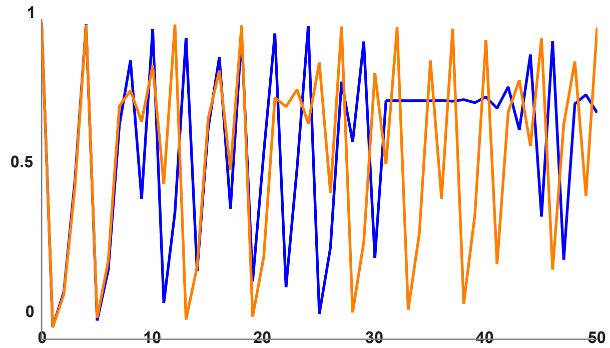


Figure 9. The dynamics of price for $p(0) = 0.99$ and $p(0) = 0.9905$

Source: evaluation of the authors

Conclusions. The price system is deterministic and not random but the dynamics gives the impression of a random series. Even more, the series is very sensitive to the initial conditions. For small change in the initial value to some other value that is very close, we observe completely different situation.

The importance of levels and rates of inflation appears most clearly when one takes a continuous view of structure and dynamics. Although a discrete view, focusing on separate events and decisions, is entirely compatible with an endogenous feedback perspective, the system dynamics approach emphasizes a continuous view. The continuous view strives to look beyond events to see the dynamic patterns underlying them. Moreover, the continuous view focuses not on discrete decisions but on the policy structure underlying decisions. Events and decisions are seen as surface phenomena that ride on an underlying tide of system structure and behavior. That underlying tide of policy structure and continuous behavior is the system dynamics focus.

References:

1. Hwarng, H., Yuan, X. (2014). Interpreting supply chain dynamics: A quasi-chaos perspective. *European Journal of Operational Research*, Vol. 233(3), pp. 566–579.
2. Oliskevych, M., Lukianenko, I. (2019). Labor Force Participation in Eastern European Countries: Nonlinear Modeling. *Journal of Economic Studies*, Vol. 46. No. 6, pp. 1258–1279.
3. Lukyanenko, I., Oliskevych, M. (2014). Labour Market in Ukraine: an Empirical Dynamic Analysis Using Error Correction Model. *Bulletin of Taras Shevchenko National University of Kyiv. Economics*, Vol. 6(159), pp. 52–58.
4. Dusza, M. (2017). Financial Chaos. *Problemy Zarzadzania*, Vol. 15(68), pp. 169–189.
5. Oliskevych, M. (2015). Processes Dynamics Asymmetry at Labour Market: Nonlinear econometric Analysis. *Actual Problems in Economics*, No. 2(164), pp. 427–436.
6. Oliskevych, M., Lukianenko, I. (2017). Structural Change and Labor Market Integration: Evidence from Ukraine. *International Journal of Economics and Financial Issues*, Vol. 7(3), pp. 501–509.
7. Anishchenko, V., Vadivasova, T. Okrokvertskhov, G., Strelkova, G. (2003). Correlation analysis of dynamical chaos. *Statistical Mechanics and its Applications*, Vol. 325(1), pp. 199–212.
8. Oliskevych, M., Tokarchuk, V. (2018). Dynamic Modelling of Nonlinearities in the Behaviour of Labour Market Indicators in Ukraine and Poland. *Economic Annals – XXI*, Vol. 169, Issue 1-2, pp. 35–39.
9. Yan, D., Ma, X., Li, T. (2019). Stability Analysis and Chaos Control of Recycling Price Game Model for Manufacturers and Retailers. *Complexity*, Vol. 2019, pp. 1–13.
10. Kozytskyi V.A., Lavreniuk S.P., Oliskevych M.O. (2004). *Osnovy matematychnoi ekonomiky. Teoriia spozhyvannia*. Lviv, LNU imeni I. Franka. Piramida, 264 s.